To what extent could a muometric positioning system be the best method of navigation in an underground environment: an experiment to determine the average positional refresh rate due to angular muon flux

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CONTENTS

I Introduction 1

II Underground positioning systems review 2
- II-A Camera based . . . 2
- II-B RFID and Zigbee . . . 2
- II-C Inertial device . . . 2
- II-D Acoustic . . . 3
- II-E Geomagnetic field . . . 3

III Experiment 3
- III-A Apparatus and setup . . . 3
- III-B Methodology . . . 3
- III-C Data analysis . . . 4
- III-D Limitations . . . 5

IV Evaluation 7

V Conclusion 7

References 8

Appendix 9

***Abstract*—With the recent development of the first muometric positioning system, it is important to determine applications in which this new system could be useful. Given the lack of viable underground positioning systems for mines, this could be a potential application for muometric positioning systems. To better evaluate this prospect, an experiment was conducted to determine the effect of a changing zenith angle on the muon flux and hence the average rate at which the positioning system can update the position of the receiver.**

The experiment found that the zenith angle had a significant impact on the muon count over 12 hours roughly following the function $cos^{1.95}(\theta)$. This was complicated, however, by the complementary effect of the reducing solid angle between detectors also serving to decrease the muon count. Perhaps the greatest limitation on the use of muometric positioning systems however was the reduction of the muon flux due to the great depths of mines. This means that impractically large detectors are required to keep a functional positional refresh rate.

These limitations suggested that a muometric positioning system would be most viable when combined in a hybrid system alongside an inertial positioning device. This would reduce the area of detector required since the muometric positioning system would only be needed to sporadically correct the inertial positioning system. Despite this, the main application of such a system would remain limited to the initial exploration of a mine before other systems could be setup rather than on individual miners due to the size of the system.

I. INTRODUCTION

In underground locations, the most common method of geographical positioning above ground, a Global Navigation Satellite System (GNSS) such as GPS, is not viable[1]. The radio waves used by these systems cannot penetrate through the ground therefore alternative methods of positioning must be used. One such example is the recently developed muometric positioning system (μPS). This technique involves detecting cosmic ray muons that pass through both a reference detector and a receiver positioned below it. By calculating the time interval between the muon being detected at the reference detector and then the receiver, the distance between them can be calculated since it is known that all muons travel at similar speeds approximately equal to the speed of light. With a network of 4 suitably positioned reference detectors, the exact location of the receiver can be deduced by the intersection of these four distances from each reference detector as shown in Figure 1.

Cosmic ray muons are particles similar to electrons, except with a greater mass, created by the decay of protons and alpha particles that enter the earth's atmosphere from space[2]. These muons can be easily detected with Geiger-Muller tubes (GM

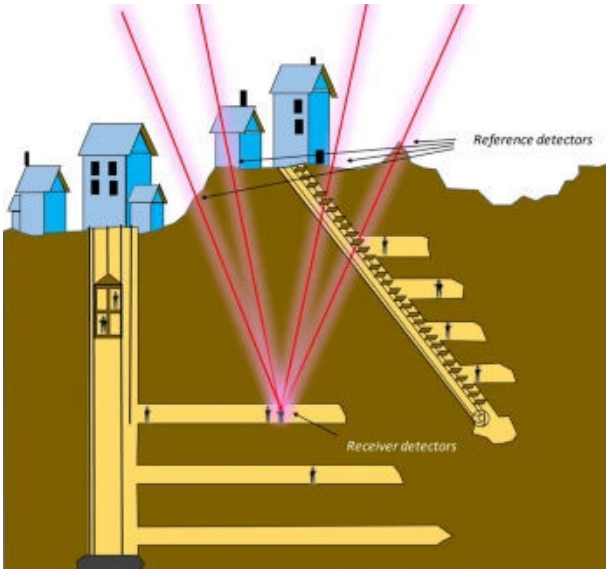


Fig. 1. A visualisation of the paths of four muons each through a reference detector above ground to the receiver detector below ground allowing the receiver's position to be determined[3].

tubes) since they are a form of ionizing radiation. Alongside this, muons are highly penetrative due to their high energy allowing them to travel large distances through solid materials before being absorbed meaning that they can still be detected underground.

The idea of a muometric positioning system (μPS) was first put into practice by Tanaka et al. (2023) who successfully created and navigated an autonomous robot around a building using this navigation technique[4]. This was particularly significant given the high accuracy of $2-25$m achieved which was already comparable to alternative systems. Furthermore, due to the highly penetrative nature of muons, the reference detectors could be placed above ground reducing how many were required. This suggested the potential viability of such a system for dynamic navigation building upon previous usages in static monitoring of seafloor displacement and volcanoes and hence how μPS could compete with other types of underground positioning systems[5][6][7]. The most notable techniques currently in use or development are camera based, RFID, inertial device, acoustic, Zigbee and geomagnetic field positioning systems. Whilst the relative advantages of these techniques have been previously compared, it must now be determined how μPS performs in comparison. Since μPS has only recently been developed, the assessment of its possible applications has been limited to underwater systems[5]. Furthermore, the testing of underground positioning systems has only occurred at shallow depths of up to 200m meaning that the possible application in mines has had little analysis given their average depth of 1000m[8][3].

In order to analyse the limitations of μPS, it is important to first understand how the range of such a system is affected by the angular muon flux. Whilst data has previously been collected on the angular muon flux, it has never been directly collected and analysed in regards to its impact on μPS[9]. At a greater angle to the vertical axis, called the zenith angle and labelled θ, the muons received have had to travel a greater path length. Since the degree of attenuation is directly proportional to the path length, the muon flux is hypothesised to be smaller at greater zenith angles. Models of the angular muon flux have previously been suggested where $I(\theta) = I(0) * cos^n(\theta)$ where $I(0)$ is the vertical muon flux, n is a constant and θ is the zenith angle[10]. The experiment presented in this paper detects the muon count at different zenith angles to see how the movement of the receiver detector away from the reference detector in a μPS setup, hence increasing the zenith angle, would affect the number of muons detected and therefore the maximum rate at which the position of the receiver can be updated. This will allow direct comparisons with other positioning systems to evaluate which would be most suitable for positioning underground.

Whilst μPS could have applications in underwater or indoor positioning, for example in warehouses, this analysis will mainly focus on the uses in underground positioning where there are fewer currently competitive alternatives[11]. In particular, there is high demand for positioning systems in the mining industry both for the tracking of workers and navigation of autonomous vehicles. This demand has especially increased since 2006 when the US PUBLIC LAW109-236 mandated that all miners had to be tracked as a safety regulation[12].

II. Underground positioning systems review

A. Camera based

This technology relies upon a visual sensor analysing features at known locations in an image and determining the distance from them. Despite its very high theoretical accuracy, it is only used rarely outside of controlled areas due to its poor performance in low lighting conditions or when objects shield its field of view.

B. RFID and Zigbee

Both RFID (Radio-frequency identification) and Zigbee positioning systems utilise networks of receivers and transmitters that send out radio waves. Systems that provide positioning with this technique involve the receiver determining the distance of the transmitter away from it by calculating the received signal strength of the radio wave. Whilst these systems are very easily scalable to multiple users by having multiple transmitters, the infrastructure requirements are very high since many receivers must be set up[13]. This is especially true for RFID where a range of only $3-5$m means many receivers are required for full coverage. In contrast, the greater range of Zigbee of up to 75m makes it more viable despite its greater power consumption and cost[4].

C. Inertial device

Inertial or dead reckoning devices are made up of a range of sensors that detect any movements of the device hence calculating the distance moved from an initial known position. The benefits of this system are that the location is calculated in real time to a very high degree of accuracy. Despite these

advantages, errors in positioning accumulate quickly over time due to imprecision in measurements meaning that its position must be reset to a known position on the order of every 10 seconds. In order to achieve this, a network of known reference points is required in a similar way to RFID and Zigbee with the large amount of infrastructure therefore required limiting the possible area over which this technique can be useful.

D. Acoustic

Acoustic positioning systems work in a very similar way to the radio based techniques using systems of receivers and transmitters except that they transmit mechanical ultrasound waves instead of radio waves. Whilst this system can achieve very high accuracy in the range of $0.01 - 3$m, it is limited in mining applications due to the large amount of objects blocking clear paths to receivers[4]. Alongside this, issues arise from multi-path propagation of the sound waves (echoes) which reach the receivers at different times and background noise from machinery which can mask the actual transmitted signals. To deal with this, high energy signals must be transmitted which increases the power use and hence cost of the system.

E. Geomagnetic field

Currently in development, the majority of geomagnetic field positioning systems function by creating a mapping or fingerprint of an area's magnetic field. Since the magnetic field of an area is fairly stable, the position of a detector in the field can be deduced by the properties of the magnetic field that it detects. Unlike the majority of other more established positioning systems, no infrastructure is required for this technique which alongside a high accuracy of < 1.2m suggests great promise[4]. The system however, is limited by there only being roughly 3 properties of the magnetic field with which to make the fingerprint suggesting that over a large area, not all locations may be unique which alongside the possibility of magnetic interference from mining machinery could make the system unusable.

III. Experiment

A. Apparatus and setup

Apparatus:
- Two Geiger-Muller tubes
- Arduino Uno processor
- Two clamp stands and bosses
- 3 Aluminium plates
- Frame on which to position the aluminium plates
- Rulers
- Plumb line

The apparatus was set up as shown in Figure 2 such that both GM tubes were attached to clamp stands and plugged in to the same Arduino Uno processor[14]. A frame was placed around the lower GM tube upon which the aluminium plates could be placed to block any direct line between the two detectors. The plumb line was hung from the clamp stand next to the higher GM tube so that the measurements could be made more easily and accurately.

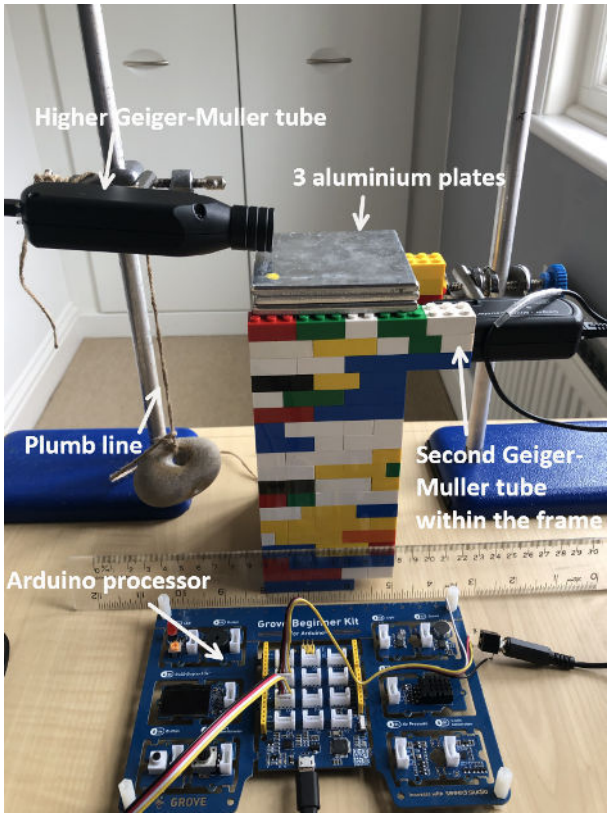


Fig. 2. Experimental setup with two GM tubes and an Arduino Uno processor.

B. Methodology

The zenith angle, θ in Figure 3, should be changed by moving the GM tube labelled number 1 in the diagram horizontally away from GM tube 2. This will increase the length x, the horizontal distance between the GM tubes, whilst h, the vertical distance between the GM tubes, remains constant. The zenith angle can be calculated by measuring x and h with a ruler and then using trigonometry. By calculating the zenith angle in this way instead of directly, the uncertainty was significantly reduced as shown in Figure 10 of the appendix. These distances were measured to a point on the centre of the actual detecting region of the GM tube. The exact length of the detecting region of the GM tube was unknown so some systematic error could have been incurred by this point not being the true centre of the detecting region. Finally, given that h remains constant whilst x changes, the solid angle, Ω, of the sky from which muons can be detected also changes. The principle of a solid angle is illustrated in Figure 4 however this must be imagined in 3 dimensions. Whilst this means that there are actually two independent variables affecting the muon flux in the experiment, this models a μPS more accurately since h would also be remaining roughly constant in an actual positioning setup. Despite this, since the value of h in this experiment is different to that which will be used in μPS, the solid angle will actually change by different proportions in this experiment to the real applications of this system introducing some inaccuracy.

This setup will have a specificity for muons due to the

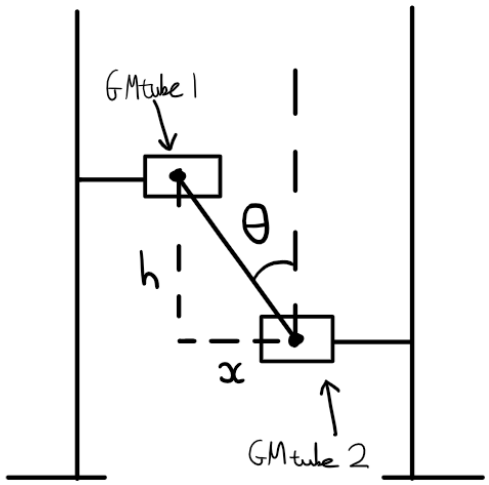


Fig. 3. A diagram of the apparatus defining the zenith angle, θ, in terms of h and x.

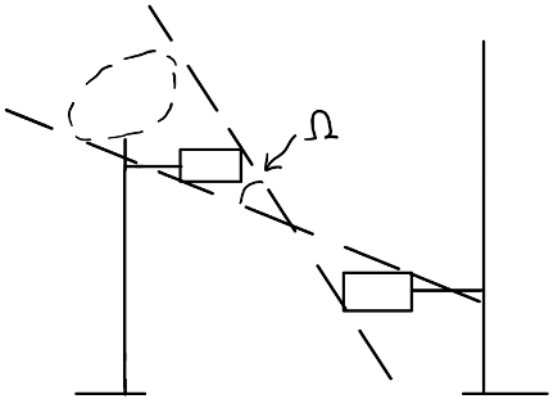


Fig. 4. A diagram defining the solid angle, Ω, for the apparatus.

use of coincidence detection. This is important since the GM tubes themselves also detect all other forms of background radiation. This technique involves accepting a radiation count as a muon only if both of the detectors are triggered within a very short time interval, in this case 1μs, meaning that the probability is very high that this coincidence was caused by the same particle travelling through both detectors. The three aluminium plates were then placed so that they blocked the direct path between the GM tubes to prevent almost any alpha or beta radiation from causing a coincidence event by travelling through both detectors since they would be absorbed by the 1.5cm layer of aluminium. Finally, since gamma radiation ionizes matter through Compton scattering, the ionization events from gamma radiation are fairly rare meaning that they don't ionize everything in their path. This therefore makes it unlikely that gamma radiation would ionize particles in both detectors. On the other hand, due to the high energy of cosmic ray muons, they ionize everything along their path allowing us to assume that all coincidence detections are caused by cosmic ray muons[9].

The GM tubes were then left running for a period of 12 hours, chosen since a longer time increases the accuracy of the results. Whilst the muon flux has been shown to vary slightly with time of day, the changes are so slight that they can be considered negligible within the accuracy of this experiment[15]. The experiments were also run on consecutive days so that there would be little variance in solar activity over the time frame since it has been shown that the activity of solar flares on the sun might be responsible for a greater variance in the Earth's cosmic ray muon flux[16]. The Arduino processor ran the code shown in Figure 11 of the appendix which was able to identify any coincidence counts as explained and increment a muon count which was displayed on an OLED display screen. The experiment was then repeated at different zenith angles.

C. Data analysis

The data collected is shown in Figure 5. To analyse the trends in the data, a graph of the muon count against the zenith angle was plotted as shown in Figure 6 where the blue crosses show the experimental data collected. This data forms a negative, non-linear trend showing that the muon count decreases as the zenith angle increases which is consistent with the proposed hypothesis. To analyse the gradient of the graph, a suggested model for the muon count is also considered where the muon count rate is given by $I(0) * cos^{1.95}(\theta)$ where θ is the zenith angle and $I(0)$ is the muon flux at a zenith angle of 0 degrees[10]. This is plotted as the orange dotted line in Figure 6. Whilst this model does simplify the geometries of the system by modelling the earth as flat, therefore is not fully accurate, it has been shown to fit experimental data well, especially at low zenith angles[9]. This model shows a similar trend in gradient to the experimental data however clearly does not fit particularly well with the muon count remaining far higher in the model than the data at a zenith angle of around 40 degrees.

This incontinuity between the experimental data and model is most likely caused by the effect of the changing solid angle. As detailed in the methodology, the solid angle was not kept constant and would in fact reduce as the horizontal distance between the GM tubes, and hence the zenith angle, were increased. This explains both why the muon count from the model remains greater and why the difference in muon count between the data and model initially increases since the model assumes that the solid angle remains equal to the starting value. The large difference between the model and experimental data in fact suggests that the solid angle has a significant effect on the muon count and that data on its effect should be collected independently. Whilst the data collected is more relevant to μPS due to the analysis of the combination of these effects, the effect of the solid angle on the muon count would likely be different when the vertical distance between the GM tubes is greater.

As the zenith angle increases and hence the muon count decreases, the average time for the μPS to be able to update your position reduces. This time is given by the average time

Zenith angle /degrees	Δzenith angle /degrees	Muon count in 12 hours /counts	Update time /mins
5.964	0.032	105	7
12.095	0.033	91	8
27.582	0.033	47	15
49.970	0.021	15	48
74.993	0.004	4	180

Fig. 5. Simplified data table from the experiment. The data of horizontal and vertical height between the detectors has been removed since these values are only required to calculate the zenith angle.

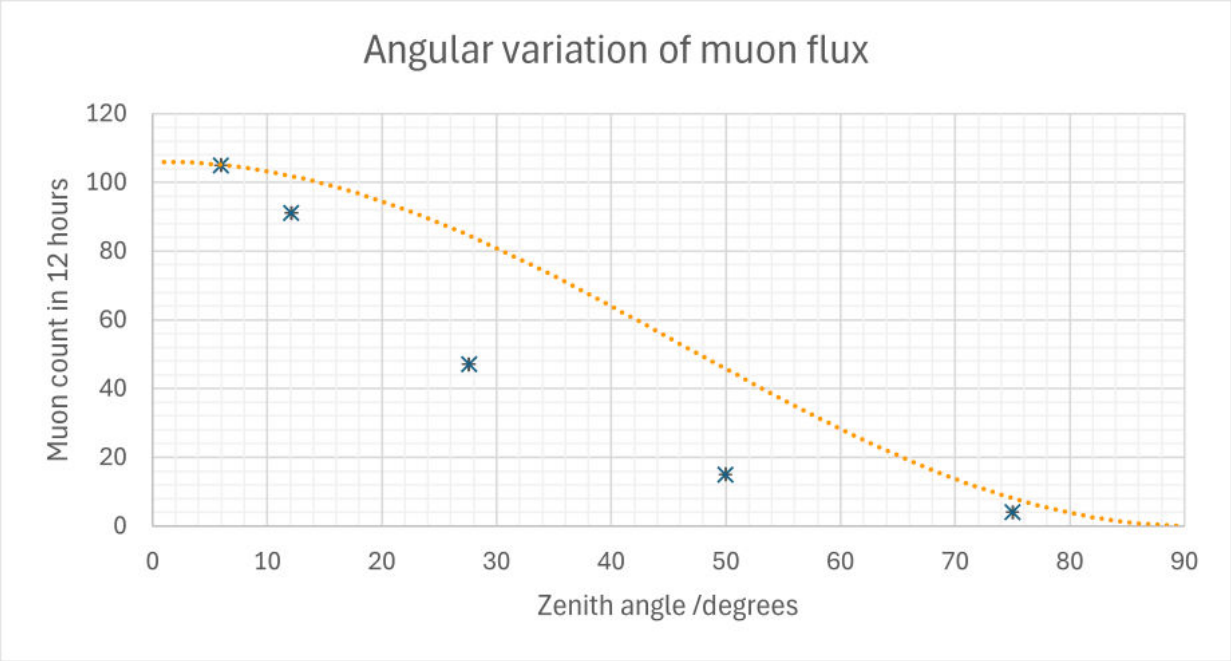


Fig. 6. Graph of the zenith angle from which muons are received against the muon count over a 12 hour period. The blue crosses show the experimental data collected whilst the orange dotted line shows a model for how the muon flux changes from Bektasoglu[10].

between two muon counts since at least one muon should pass through each reference detector to the receiver in this period. A graph of the positional update time against zenith angle is shown in Figure 7. To determine a maximum possible horizontal range for a μPS, we shall set a limit to the minimum muon count of half of the original count to ensure that the positional update time doesn't degrade too significantly as the detector moves horizontally. This in effect specifies a zenith angle beyond which the muon count is considered too low. From the experimental data collected, this zenith angle is approximately 26 degrees. Using the average mine depth of 1000m at which a μPS might operate, the maximum horizontal range of the system would therefore be 488m. These values can be easily calculated with the method shown in Figure 8.

D. Limitations

One of the major limitations faced in the experiment was reducing the time interval over which a coincidence event was considered to be a muon. An initial experimental setup was limited to a minimum time interval of 20ms by the hardware which led to a large random systematic error due to spurious coincidence events[17]. These were caused when each GM tube detected a separate particle of ionizing radiation within the same time interval of 20ms leading to 65 spurious counts in 12 hours. The dramatic effect on the data is shown in a graph from previous experimental results in Figure 12 in the appendix. The impact is particularly evident from the muon count remaining at 139 over 24 hours even at a zenith angle of 90 degrees when it should be 0. By bypassing the official data collection equipment with an Arduino processor, the time interval was reduced to the 1μs used both in this experiment and by Tanaka et al. in the first functioning μPS[4]. This led to an average of only 0.003 spurious counts in 12 hours as calculated in Figure 13. Whilst it was relatively simple to reduce this time interval to 1μs in this experiment since the GM tubes were connected to the same Arduino processor and hence the same clock, it is far more difficult when the reference and receiver detectors are over 1000m away from each other. It is clearly not viable to have wired connections from both of these detectors to the same clock hence wireless communication must be used with a separate clock for each detector. This creates a hardware limitation to the system

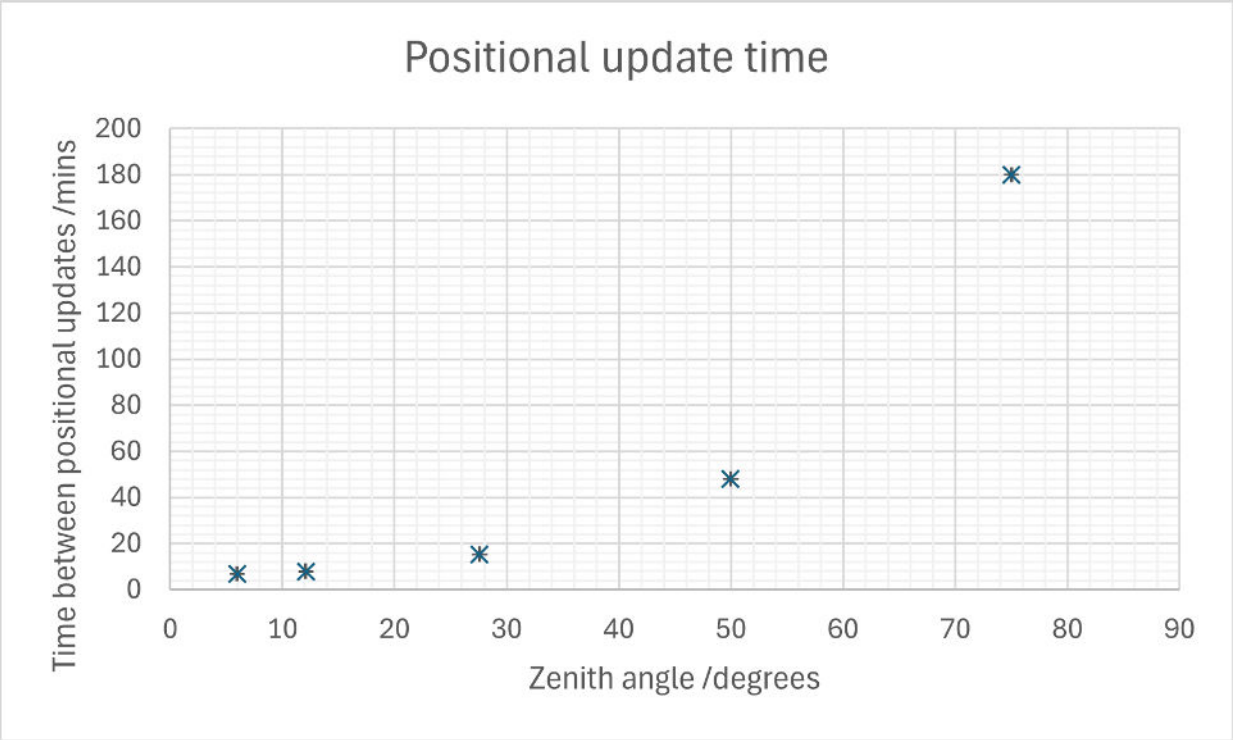


Fig. 7. A graph of the zenith angle against the average time between coincidence counts. This time corresponds to the average time interval between which the position produced by the μPS is updated.

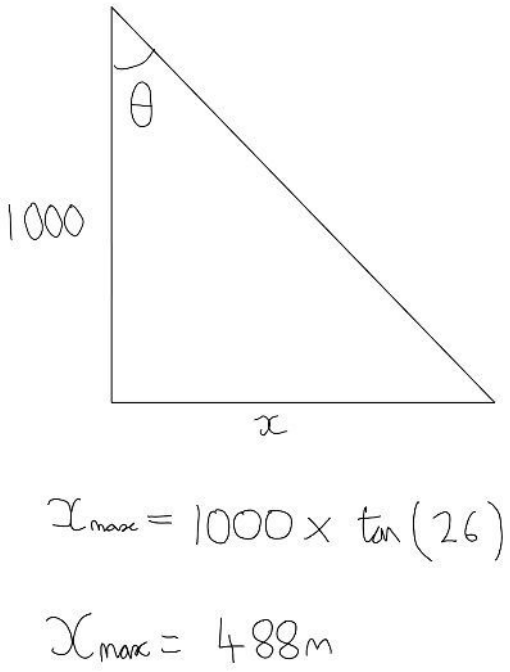


Fig. 8. This diagram shows how the zenith angle affects the maximum horizontal range of μPS.

with expensive clocks, such as Cs oscillators, required to minimise fluctuations in the clocks and hence allow this higher specificity to muons.

Another major limitation is that the positional update rate will always be limited by the environmental muon flux. The muon flux has been recorded as approximately $10,000$ muons per minute, per metre squared at sea level which is relatively low[18]. The muon count from a detector can be increased by either increasing the time of measurement, increasing the area of the detectors, increasing the solid angle between the detectors or decreasing the zenith angle. Given the low areas of the GM tubes used in this experiment, the experiment had to run for 12 hours to detect enough muons that the uncertainty due to the randomness of radioactivity was suitably minimised. For μPS, since we want to minimise the time that it takes to detect a muon, and the zenith angle and solid angle cannot be controlled due to the movement of one of the detectors, the only way to ensure a low positional update rate is to have detectors with large areas. The need for detectors on the order of a metre squared (the area used by Tanaka et al.) poses a severe limitation to the use of a μPS since this would be impractical to place upon a single person and may limit the technique's applications to larger machinery or autonomous systems.

For the potential application of μPS in mines, the ability for the system to work at large depths is clearly important. As ore deposits are depleted, functioning mines are gradually getting deeper with most mines now around 1000m with some coal mines reaching up to 1500m and certain precious metal mines even reaching 4500m[8][19]. Data collected on the variation of muon flux with depth has shown that at 1390m, the muon flux is 12 muons per metre squared per day and at 2000m, it reduces to 0.27 muons per metre squared per day[20][21].

Positioning system type	Accuracy /m	Horizontal Range /m	Update time	Maximum operational depth	Size of user element
GNSS	10-200	Unlimited	1Hz	0m	Small
MuPS	2-25	488	0.01Hz	100m	Large
Camera based	0.1	Unlimited	Real time	Unlimited	Fairly large
Dead reckoning	Errors accumulate	10	Real time	Unlimited	Small
RFID	10-30	3-5	High	Unlimited	Small
Acoustic	0.01-3	10	Medium	Unlimited	Small
ZigBee	10-200	1-75	High	Unlimited	Small
Magnetic field	<1.2	Pre-determined	High	Unlimited	Small

Fig. 9. Table comparing the different types of underground positioning systems over key requirements[4]. GNSS is included so that comparisons can be made to the performance expected of overground positioning systems.

This is in contrast to the surface muon flux of 14 million muons per metre squared per day. This clearly poses a great challenge since the low muon flux would limit the positional update time of μPS as well as the precision of the positioning itself. The positional update time of 2 hours at 1390m with a metre squared detector is clearly not viable hence even larger detectors would have to be used presenting further setbacks for both mobility and cost of the system. Given that this is around the average mine depth, the low muon flux perhaps poses the greatest limitation to μPS in mines with the system likely not being viable at all for the deepest precious metal mines.

IV. Evaluation

Whilst advantages of μPS include its low infrastructure requirements and high penetration allowing it to be unaffected by obstructions, it is clear that μPS has limitations, some of which are compared to those of other underground positioning systems in Figure 9. The performance of GNSS is given on the first row to allow baseline comparisons however GNSS can not be used underground as previously explained. In order to determine the most suited system for applications in mining, priorities of the requirements must first be decided. Given that mines are predominantly made up of systems of tunnels with few open spaces in which more precise positioning might be required to locate an object or person, accuracy of the positioning system is not the greatest concern. Whilst other positioning systems can provide greater accuracies than μPS as shown in the table, an accuracy of roughly 10m as achieved by the μPS created by Tanaka et al. would be sufficient. A far higher priority for this application would be the horizontal range of the system and hence the scalability to cover a large area since mines can be very large. Here μPS has a clear advantage over alternatives such as RFID, Zigbee and acoustic positioning systems which would require many more receivers in a reference network due to their lower ranges of transmission. This large number of receivers would likely make costs greater than those of μPS which have been estimated at $\$84,000$ for the reference stations and $\$15,000$ for each receiver[3].

Another key consideration is the size of the user element of the positioning system since mines have restricted space. This is perhaps the most significant disadvantage of μPS given that it requires a large area for the receiver detector. This makes the system unfeasible for the tracking of individual miners due to the lack of portability and weight of the system. An area greater than a metre squared would therefore be required for the detector so that the muon flux remains high enough to give a reasonable positional update time. Whilst this experiment has analysed the effect of the zenith angle on the muon count, which has a large impact on the horizontal range of the system, the horizontal range does not seem to be the limiting factor for μPS. Instead, the limited muon count due to the 1000m of rock above most mines poses a greater challenge. To overcome this, there is the possibility of using a hybrid positioning system made up of both μPS with a slower positioning update time, alongside an inertial positioning device that provides real time positioning. These techniques would complement each other well since inertial positioning requires resetting on the order of every 10 seconds, a service which could be provided by the μPS. Whilst this could help the issue of the low positional update time of μPS, a lack of robustness of the detector to knocks could still limit its uses to more static mining vehicles and equipment rather than to track autonomous machines[22]. A hybrid positioning system including μPS could perhaps be the most viable option for the initial exploration of caves or new mines before a large reference system could be set up as required for the majority of other positioning systems such as geomagnetic, RFID or Zigbee. Since μPS is one of the only viable positioning methods here, the low positional update rate may have to be accepted if tracking of objects or miners is required at the early stages of mine exploration.

V. Conclusion

This experiment has served to highlight some of the intrinsic limitations of μPS including the large area required for its receiver device and the highly precise and expensive hardware such as clocks with a precision greater than 1μs needed. The experiment demonstrated the effect of the zenith angle on the muon count and hence the maximum horizontal range of the system however it was in fact determined that the solid angle between the detectors might have just as significant an impact. Before more money is invested into increasing the precision of μPS, it would be important to fully analyse the implications of the solid angle on the muon count and hence

maximum horizontal range of the system when the vertical distance between detectors is large.

After comparing the properties of μPS to current alternative underground positioning systems used in mines, it was found that a hybrid system combining μPS with inertial positioning could be advantageous for certain specific applications such as positioning in recently excavated areas of a mine since μPS requires no pre-existing hardware within the mine. The low muon flux at the great depths of most modern mines means that any μPS would require a receiver area too large to be feasible for use by individual miners. Finally, whilst this paper has analysed the potential uses of μPS for mining applications, it is important that the advantages of this positioning technique are also considered for other applications including those underwater and indoor where the muon flux would be higher since it has clearly been shown that μPS does have certain advantages over other alternative positioning systems.

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APPENDIX

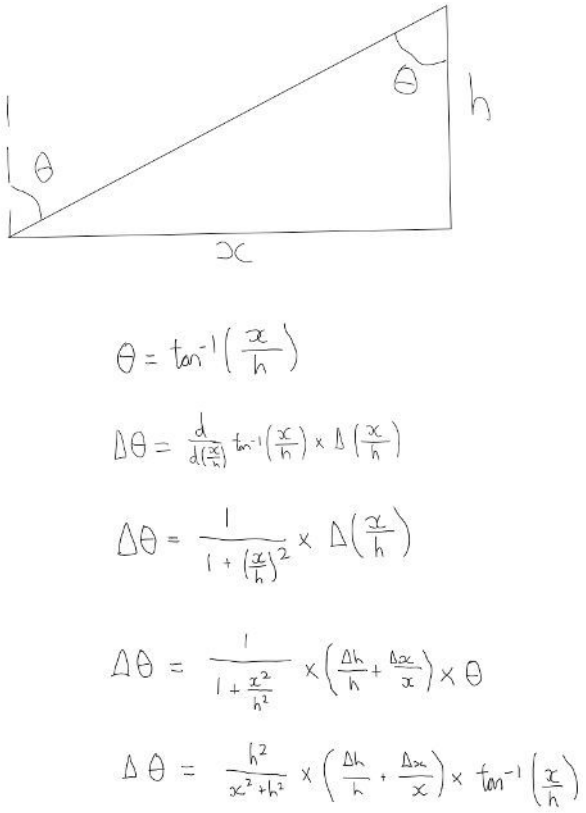


Fig. 10. This diagram shows how to calculate the uncertainty in the zenith angle from the measurements of x and h.

arduino_muon_processor.ino

```
#include <time.h>
#include <UkesfSixthFormers.h>

const int GMtube1 = 2;
const int GMtube2 = 3;
int muon_count = 0;
bool GMstate1;
bool GMstate2;
Display display;
String muon_count_display;

void setup() {
  pinMode(GMtube1, INPUT);
  pinMode(GMtube2, INPUT);
  Serial.begin(9600);
  display.begin();
}

void loop() {
  long start = micros();
  long current = start;
  while ((current-start) <= 43200000000) {
    GMstate1 = digitalRead(GMtube1);
    GMstate2 = digitalRead(GMtube2);

    if (GMstate1 == HIGH) {

      long start = micros();
      long current = start;
      while ((current-start) <= 1){
        GMstate2 = digitalRead(GMtube2);
        if (GMstate2 == HIGH){
          muon_count += 1;
          muon_count_display = String(muon_count);
          display.printBlackLarge(5, 2, muon_count_display);
          break;
        }
        current = micros();
      }
    }

    if (GMstate2 == HIGH) {
      long start = micros();
      long current = start;

      while ((current-start) <= 1){
        GMstate1 = digitalRead(GMtube1);
        if (GMstate1 == HIGH){
          muon_count += 1;
          muon_count_display = String(muon_count);
          display.printBlackLarge(5, 2, muon_count_display);
          break;
        }
        current = micros();
      }
    }
    current = micros();
  }
  Serial.println(muon_count);
  delay(100000);
}
```

Fig. 11. This is the code that the Arduino processor ran to identify coincidence counts in real time. It is written in the Arduino language which is a form of C++.

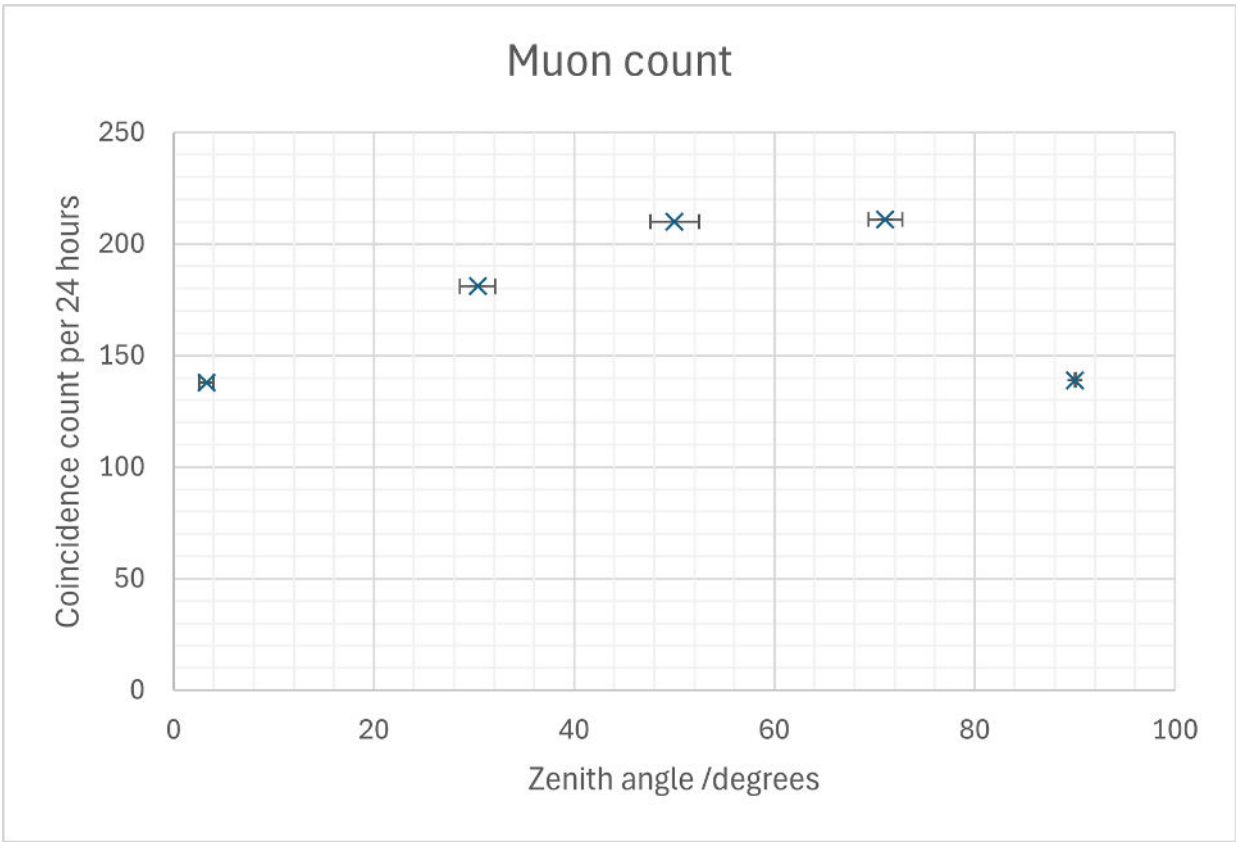


Fig. 12. A graph of the zenith angle against the muon count from the first experimental setup using the built in processing hardware from the GM tubes. This graph is clearly very different to the results from the second experiment with little to no trend and a random systematic error from spurious muon counts.

Coincidence time interval = $1\mu s$

Background radiation count rate = 16.51 counts min^{-1}

Probability of a detection in $1\mu s$: 2.7517×10^{-7}

Using the poisson distribution:

$X \sim P(2.75 \times 10^{-7})$

$(1 - p(x=0))^2 = 7.5717 \times 10^{-14}$

Number of spurious coincidences in 12 hours = $7.517 \times 10^{-14} \times 10^6 \times 60 \times 60 \times 12$

$= 0.003271$ counts

Fig. 13. How to calculate the number of spurious muon counts over the 12 hour period. This uses the poisson distribution and the background radiation count rate which was separately recorded over a 24 hour period.